Many-body magic: tensor network states and gauge theories

MBQM workshop@Abu Dhabi, 20.11.24

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ICTP, Trieste

Joint work with **T. Chanda**, **M. Frau**, **P. Tarabunga, E. Tirrito**, and collaborations with M. C. Bañuls, **M. Collura,** P. Falcao, R. Fazio, G. Fux, A. Hamma, G. Lami, L. Leone, S. Oliviero, and J. Zakrzewski

References: PRXQ 4, 040317 (2023), PRA 109, L040401 (2024), PRL 133, 010601 (2024), PRB 110, 045101 (2024), 2409.01789 and 2411.11720.

Nice to be at this workshop

Don’t have to justify the name magic

Don’t have to discuss why that’s cool

Don’t have to spend 5 minutes reviewing Clifford group :)

Why shall many-body theorists bother about magic?

"Real" cost of quantum computing

Opportunities to learn about quantum phenomena
(classification, probing etc.)

Opportunities to harness complexity and leverage this
to perform better/new simulations and theory

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Opportunities to learn about quantum phenomena (classification, probing etc.)

Entanglement: topological orders, criticality

Opportunities to harness complexity and leverage this to perform better/new simulations and theory

Entanglement: tensor network simulations

Few things I'd like to discuss today

Tensor network and magic: how to compute it

Compute

PRXQ 4, 040317 (2023), PRL 133, 010601 (2024)

Magic in lattice gauge theory: is there something interesting?

Learn

2409.01789

Can magic lead us to understand the deeper structure of many-body Hilbert spaces?

New methods?

PRB 110, 045101 (2024) and 2411.11720.

Step 0: what and how we compute

Many meaningful witnesses and measures, interesting quantities

Robustness, mana, min-relative entropy of magic, Bell magic

Our target: stabilizer Renyi entropies (Alioscia's talk)

$$M_n(\rho) = \frac{1}{1-n} \log \sum_{P \in \mathcal{P}_N} \frac{|\mathrm{Tr}(\rho P)|^{2n}}{d^N}$$

Leone, Oliviero & Hamma, PRL 128, 050402 (2022)

Stabilizer Renyi entropies

$$M_n(\rho) = \frac{1}{1-n} \log \sum_{P \in \mathcal{P}_N} \frac{|\mathrm{Tr}(\rho P)|^{2n}}{d^N}$$

- Formulated as expectation values of string operators
- Satisfies properties of measures*
- Probabilistic interpretation - *pure gold*
- Still, requires to measure of measurements $\propto 4^N$

Leone, Oliviero & Hamma, PRL 128, 050402 (2022). See also discussion in Piroli and Haug, Quantum 2023.

How can we compute Stabilizer entropies?

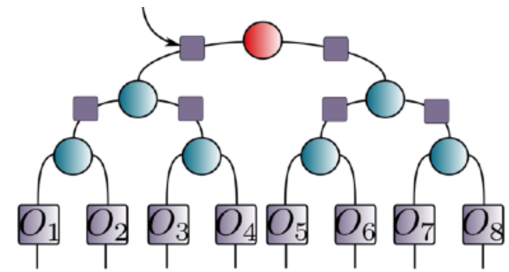
$$M_n(\rho) = \frac{1}{1-n} \log \sum_{P \in \mathcal{P}_N} \frac{|\mathrm{Tr}(\rho P)|^{2n}}{d^N}$$

We focus on one-dimensional (+small 2D) systems

- Tons of physics: conformal criticality, topology, frustration, dynamics
- Relevance to synthetic matter experiments
- Paradigms for quantum computing blocks
- Solid theory framework: tensor networks

Stabilizer entropies and tensor networks

$$M_n(\rho) = \frac{1}{1-n} \log \sum_{P \in \mathcal{P}_N} \frac{|\mathrm{Tr}(\rho P)|^{2n}}{d^N}$$



Exact approaches

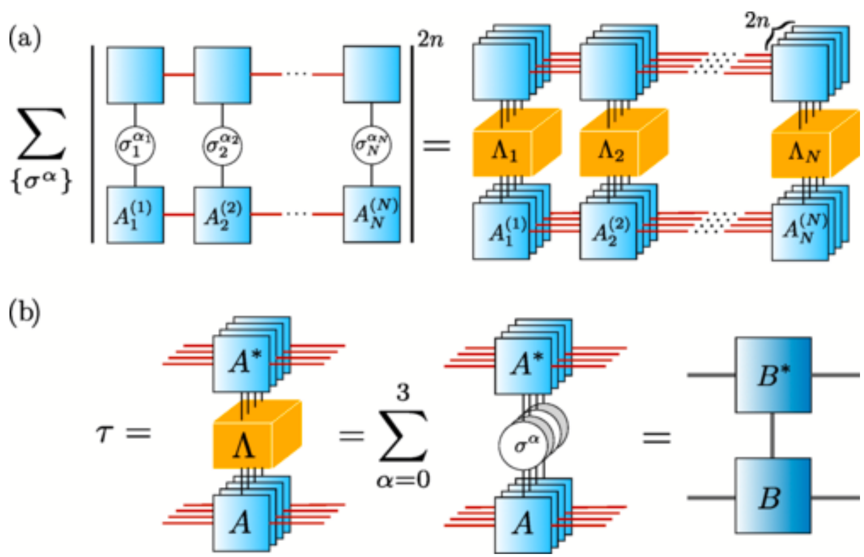
Haug, & Piroli, PRB 107, 035148 (2023);
Tarabunga et al., PRL 133, 010601 (2024).

Stochastic

Lami & Collura, PRL ’23; Piroli and Haug, Quantum ’23;
Tarabunga et al., PRXQ 4, 040317 (2023)

Insight: special structure of matrix product states might work!

Basic idea: use a replicated matrix product state (MPS)



Haug, & Piroli, PRB 107, 035148 (2023).

- Very elegant, yields *exact* SRE
- While efficient, cost is

$$O(N\chi^{6n})$$

Demonstrated only for Ising model at $\chi = 12$

MPS: states with finite (area law) entanglement between partitions

New insight: stochastic method

In some cases, direct sampling is possible!

Lami & Collura, PRL ’23; Piroli and Haug, Quantum ‘23

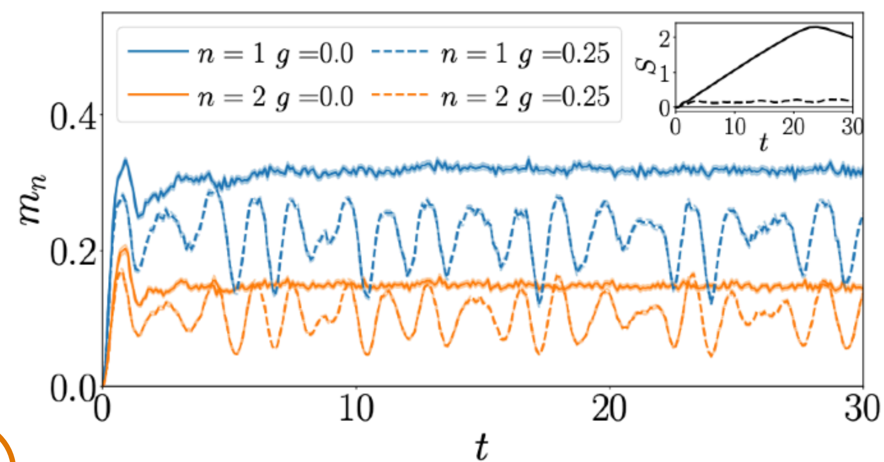
For the case of connected partitions and OBC, with MPS

$$O(N\chi^3)$$

Much better scaling (and efficient for n=1)!

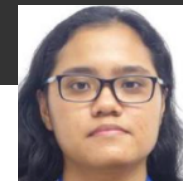
However:

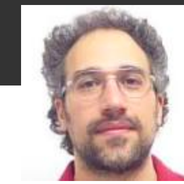
- hard to scale to 2D, PBC
- disconnected partitions
- n>2

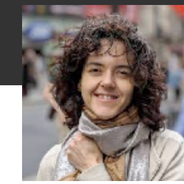


$$\chi = 32, L = 40$$

Pauli MPS



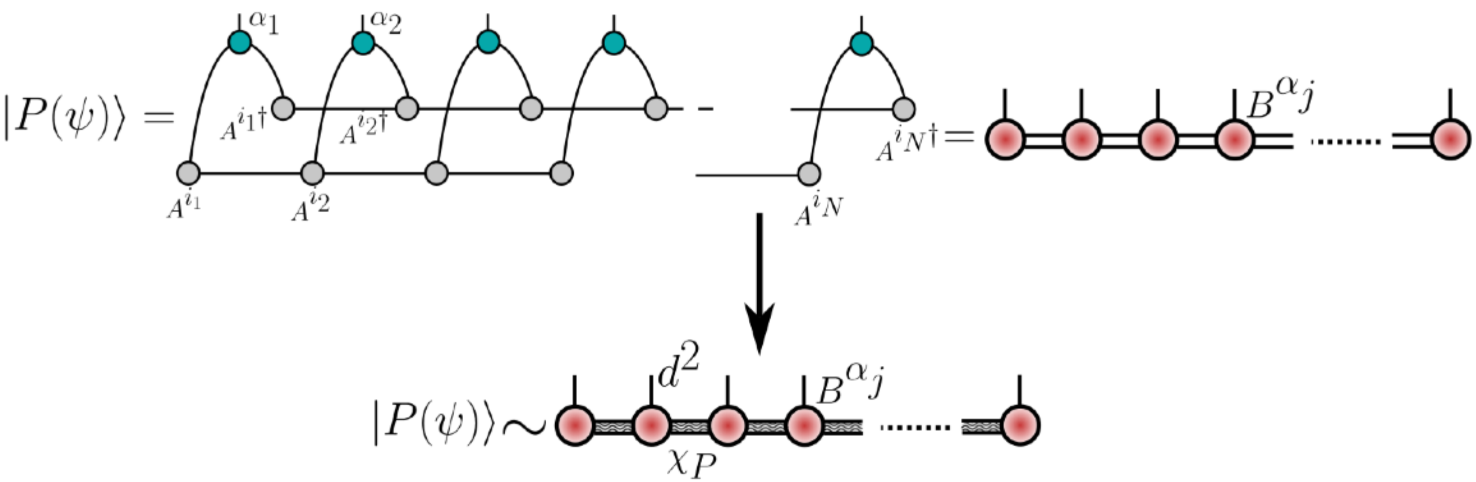




Matrix product state in a replicated Pauli basis - Pauli MPS

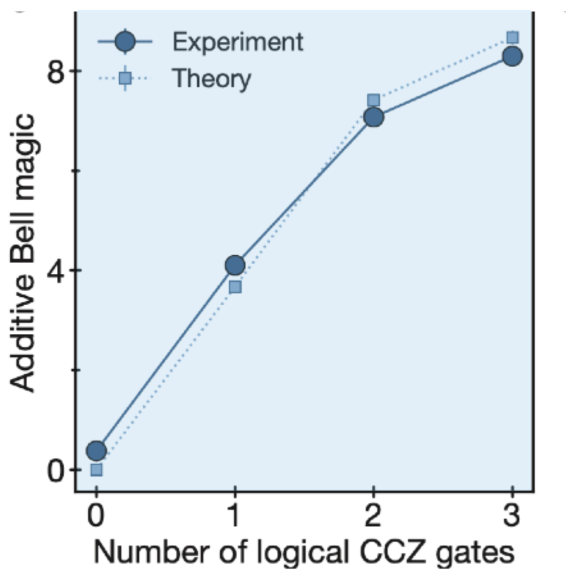
PRL 133, 010601 (2024)

- No sampling errors
- Exact control of truncation errors
- Modest scaling $O(\chi^4)$

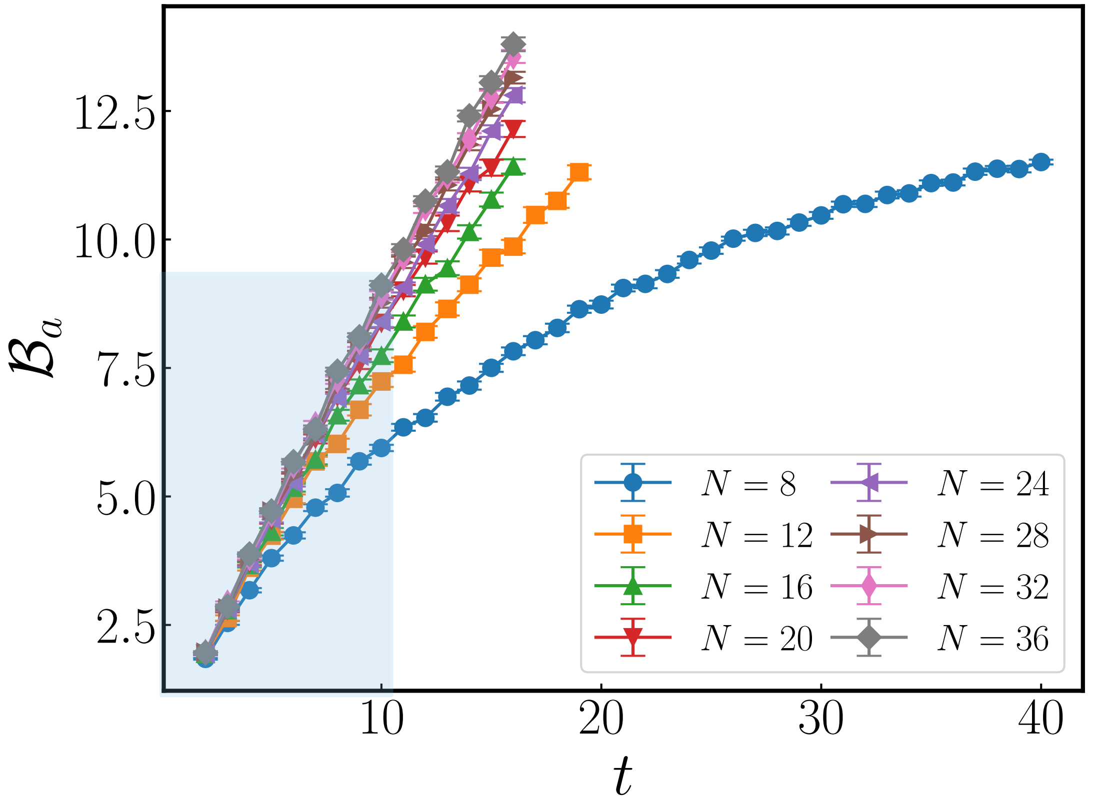


Pauli MPS

Can be used to measure other magic-related quantities
(e.g., Bell Magic - Haug and Kim, 2023)



Lukin’s group, Nature 2023
Largest chuck of ‘magic’ matter measured to date



Going around: Pauli Markov chains

$$M^{(n)}(|\psi_N\rangle) = \frac{1}{1-n}\log\left\{\sum_{P\in P_N}\frac{Tr(\rho P)^{2n}}{2^N}\right\}$$

Sample not states, but the **distribution of Pauli strings**, with importance sampling!

Pauli-Markov chains

NB: straightforwardly applicable to experiments, albeit role of statistical errors unclear. Also applicable to other numerical methods

Our tool: Pauli Markov chains

Algorithm:

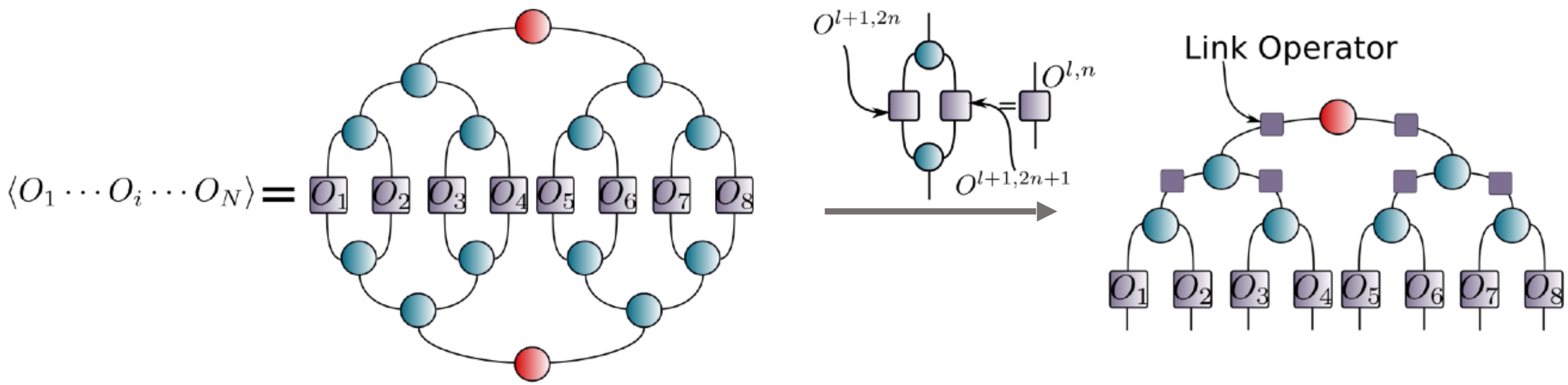
Input: a quantum state ρ and number of sampling N_S

1: Initialize the Pauli string P.
2: Compute $\mathrm{Tr}(\rho P)$ and Π_P.
3: **for** $(i = 1;\ i \leq N_S;\ i++)$ **do**
4: Propose a candidate Pauli string P'.
5: Compute $\mathrm{Tr}(\rho P')$ and $\Pi_{P'}$.
6: Accept the move with probability: $\min\left(1, \frac{\Pi_{P'}}{\Pi_P}\right)$.
7: Measure the estimators.
8: **end for**

Output: a Markov chain of P with probability Π_P.

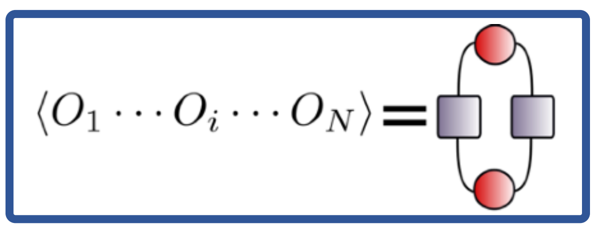
Key elements to address:
- Reliability of estimators
- Tensor contractions
- Memory effects

Tensor contractions: example of trees



$$O(\log(N)\chi^4)$$

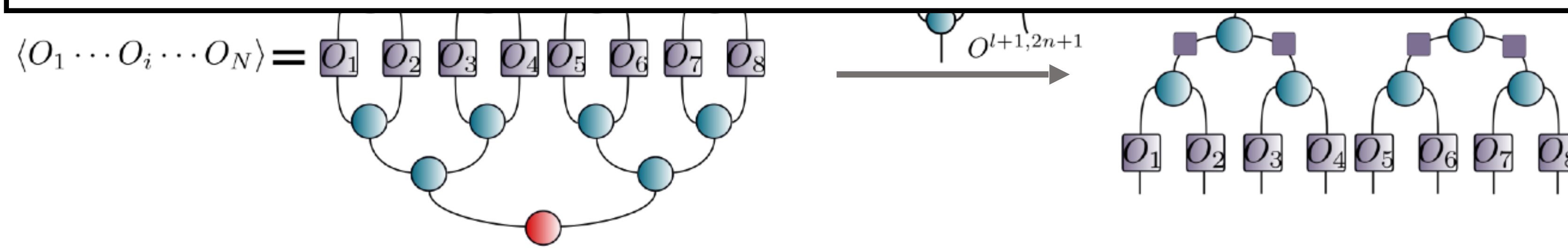
For local (e.g., 1- or 2-sites) updates, can be contracted very efficiently!!!



- arbitrary partitions
- PBC
- dimension plays very little role
- Stochastic
- Easily extended to MPS, PEPS etc.

Tensor contractions: exampl

You don't want me to go over this…



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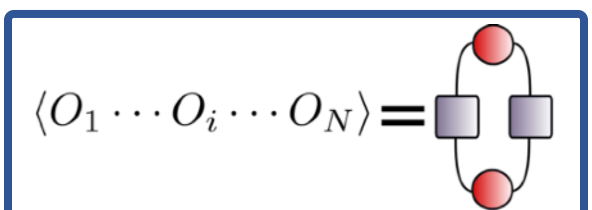

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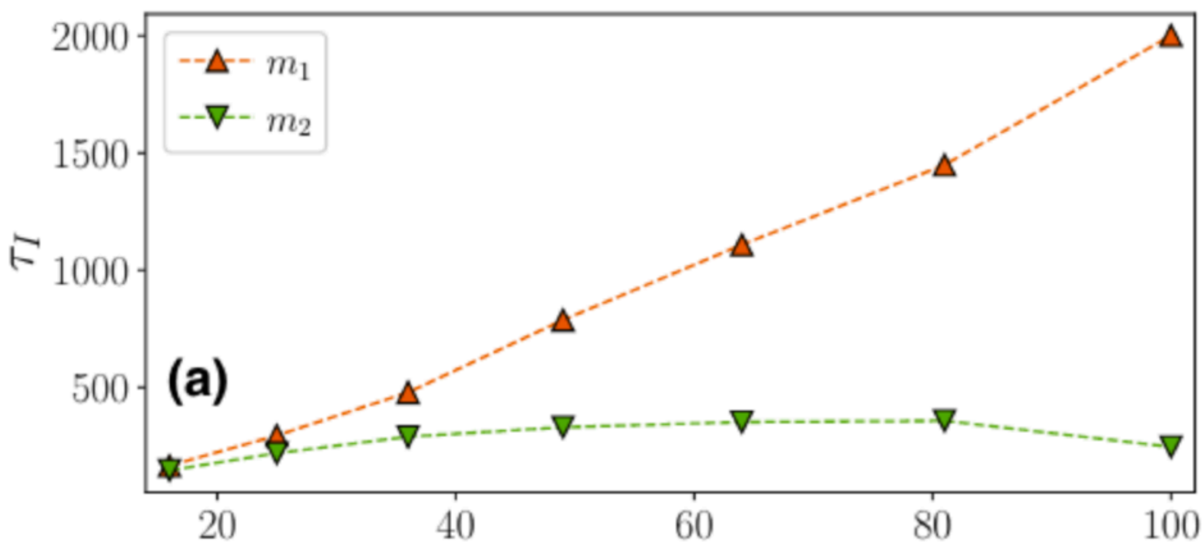
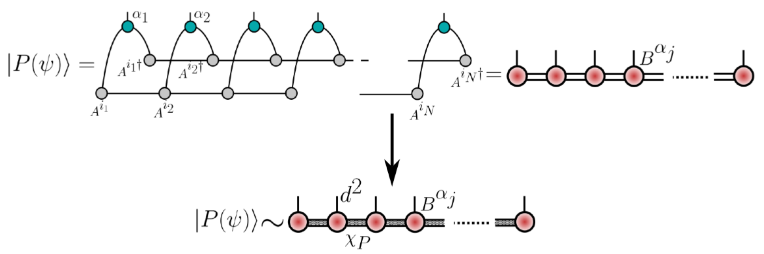
Memory effects: Autocorrelations

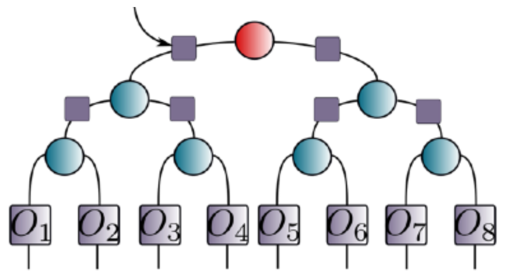


2D Ising LGT at h=3, N=LxL

$$\chi = 60, N_S = 10^6$$

Method development: summary





We can now do trustworthy computations of magic for relatively large volumes and entangled states

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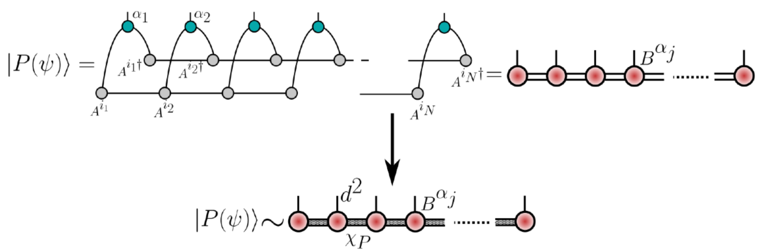

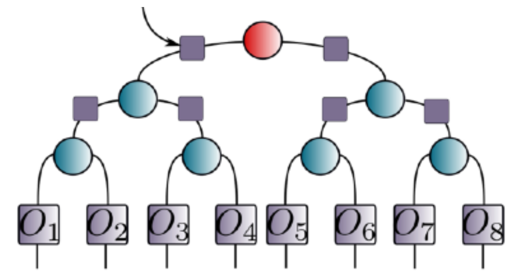

We can now do trustworthy computations of magic for relatively large volumes and entangled states

Now, we can look at some physics!! *This was not possible before 2022* - a great example of quantum info and many-body synergy!

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PRXQ 4, 040317 (2023),
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Magic in lattice gauge theory: is there something interesting?

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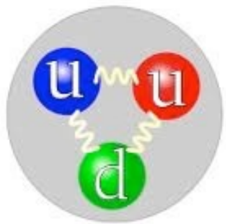
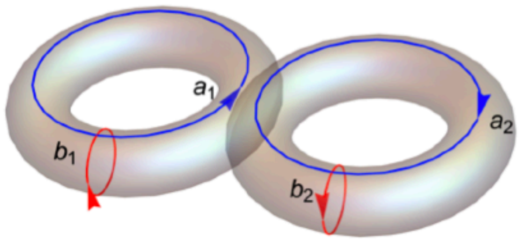
Why the hell gauge theories???

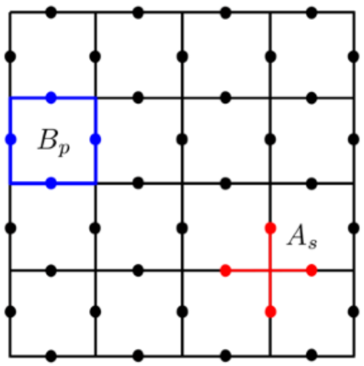
Fundamental interactions



Topological matter



Actually, epitome models for error correction!!



First studies in LGTs: upper bounds

In HEP, people are very interested in complexity of gauge theory simulations

							Schwinger bosons	
x	η	L	t/a_s	Δ	$\alpha_{\text{Trot.}}$	$\alpha_{\text{Newt.}}$	Qubits	T gates
1	4	100	1	0.01	90%	9%	2626	8.19713×10^{11}
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2D SU(2), state preparation

Davoudi, Shaw, Stryker, Quantum 2024

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2D SU(2), state preparation

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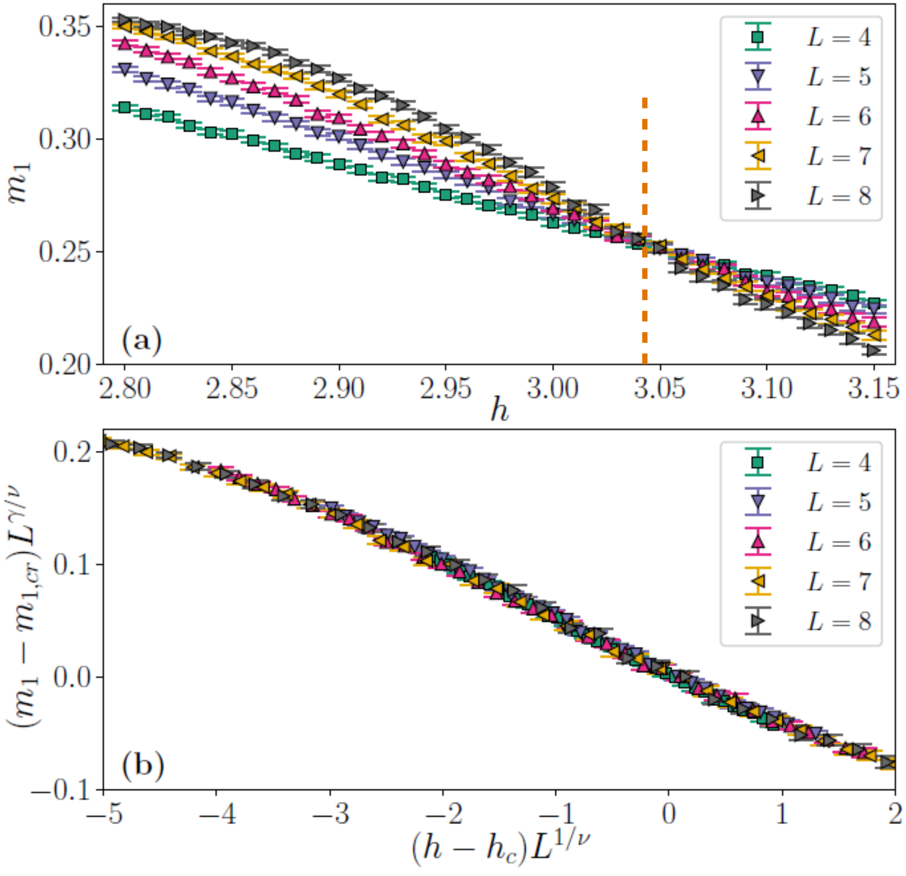
Confinement/deconfinement transition

$$H_{Z_2-\text{Gauge}} = -h\sum_{\boxdot}\prod_{i\in\boxdot}\tau_i^x - \sum_i \tau_i^z$$



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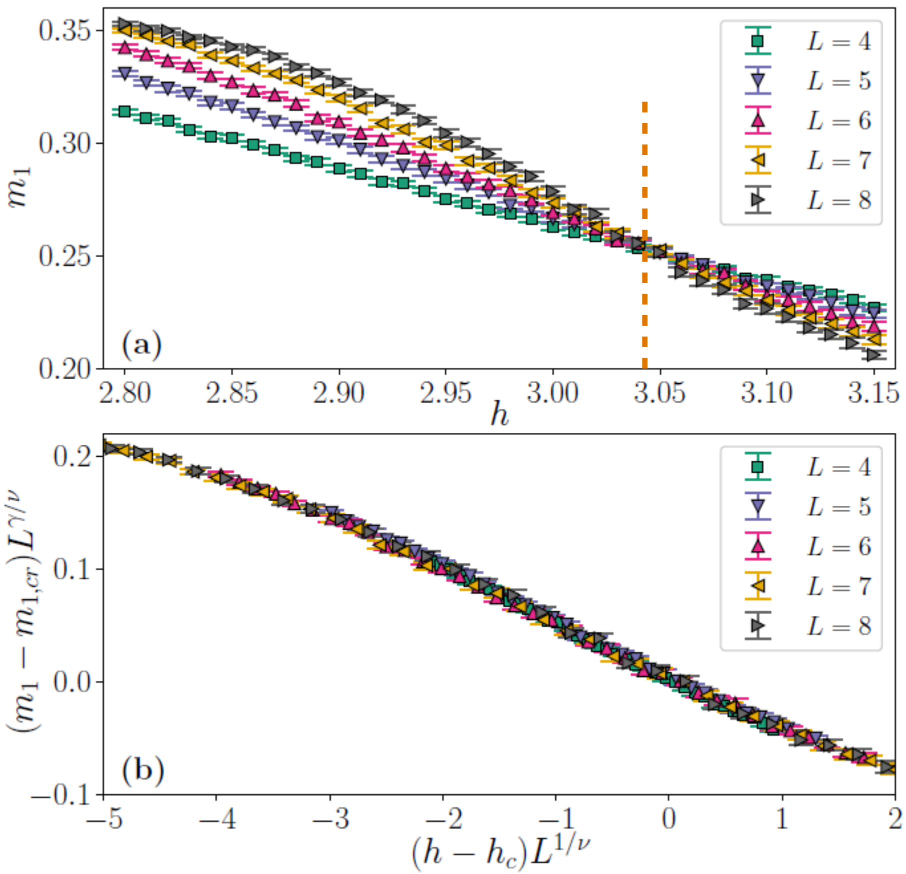

- very different from 1D: magic displays crossing at criticality

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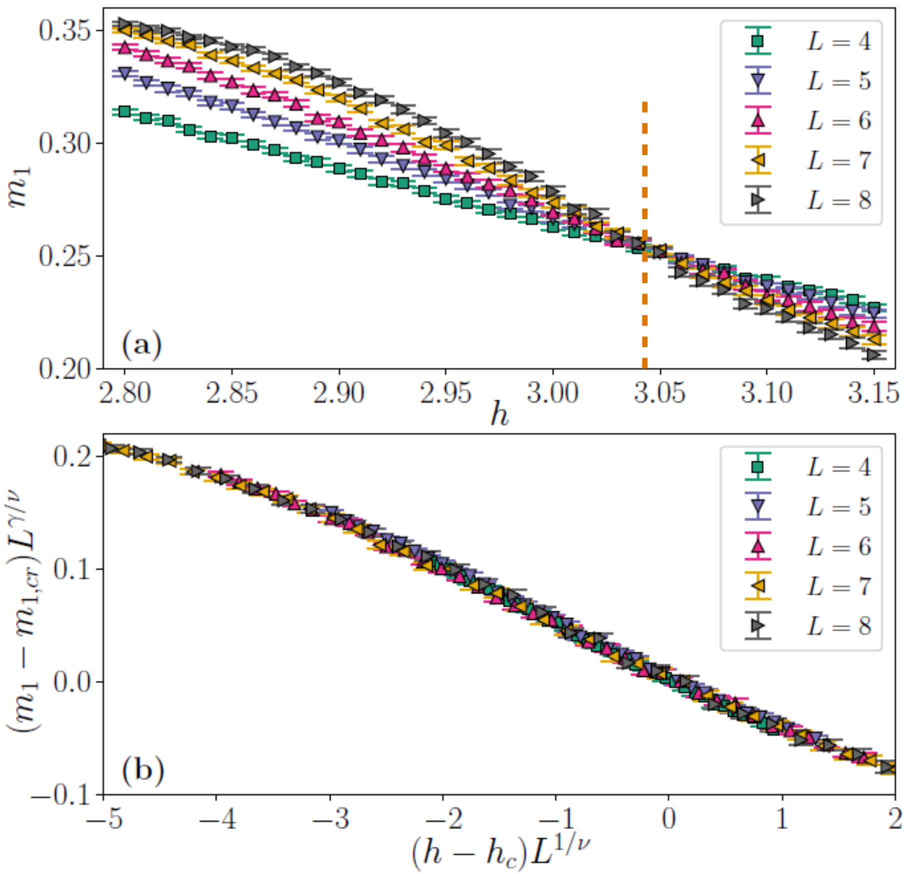

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- very impressive collapse scaling!

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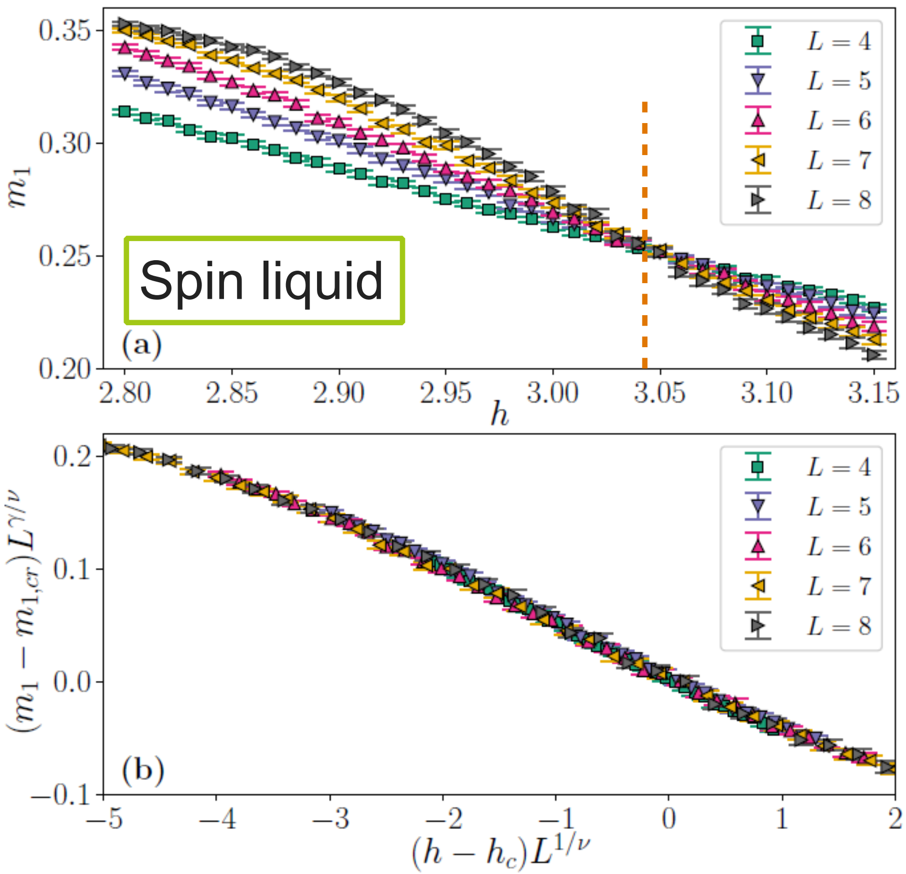

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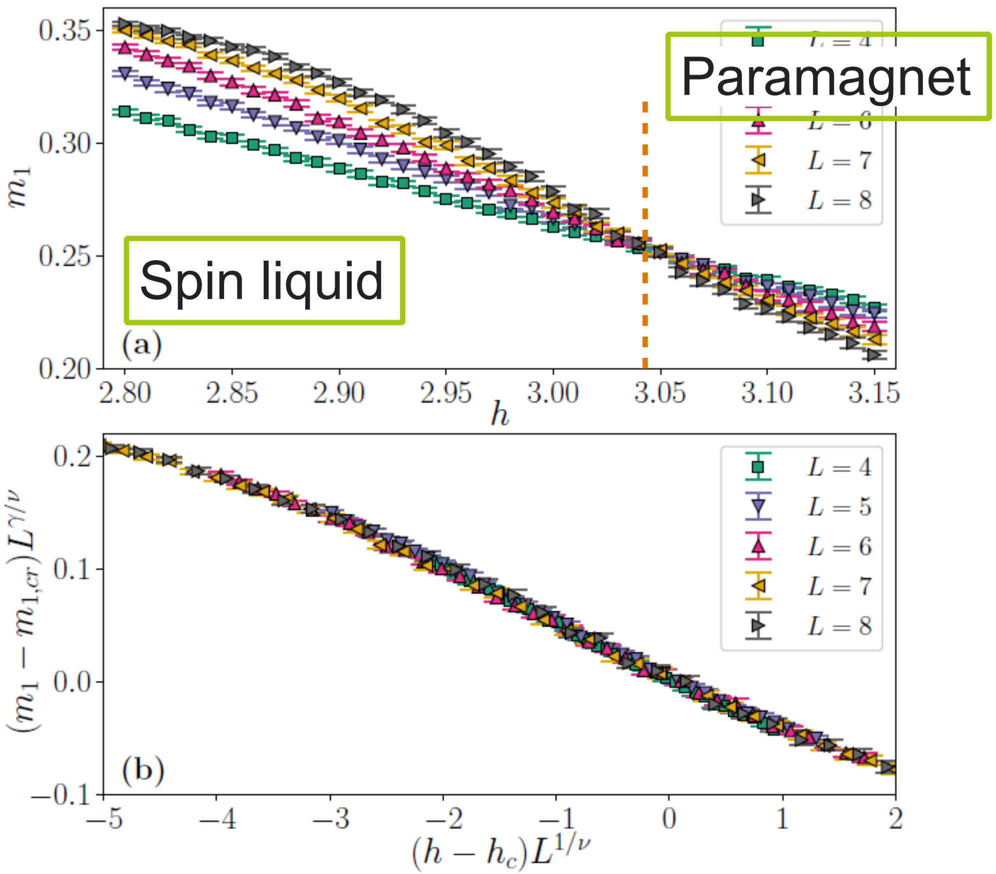

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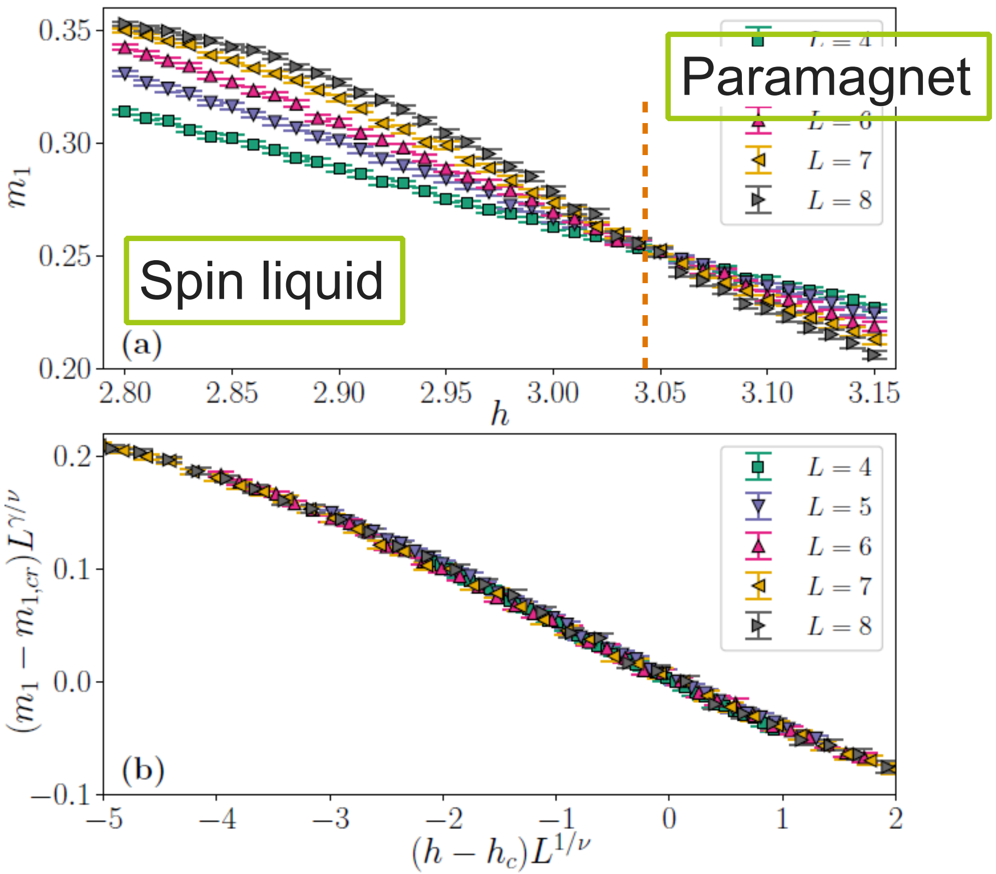

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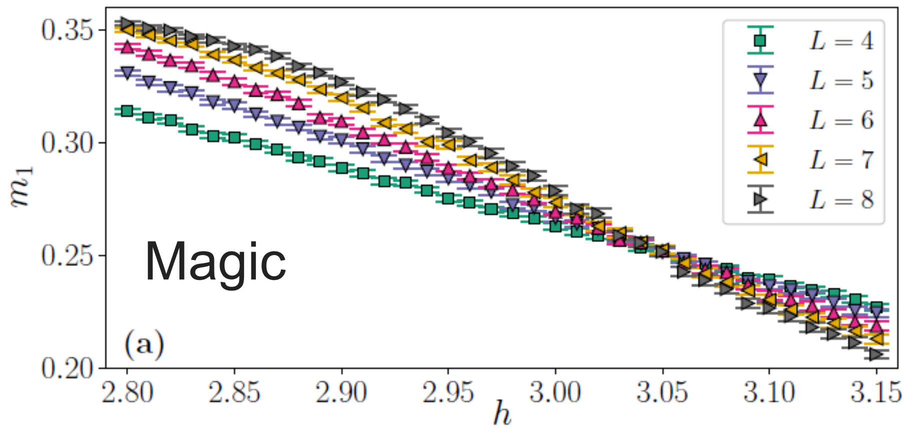
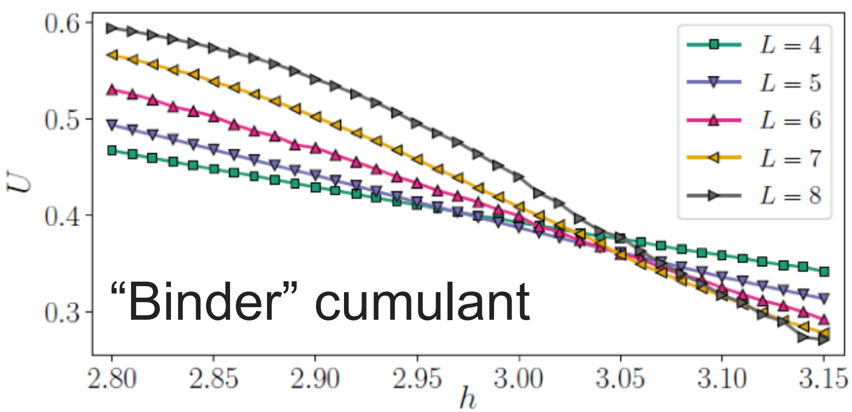
See also recent QMC results by Clark and Liu

Lattice gauge theories





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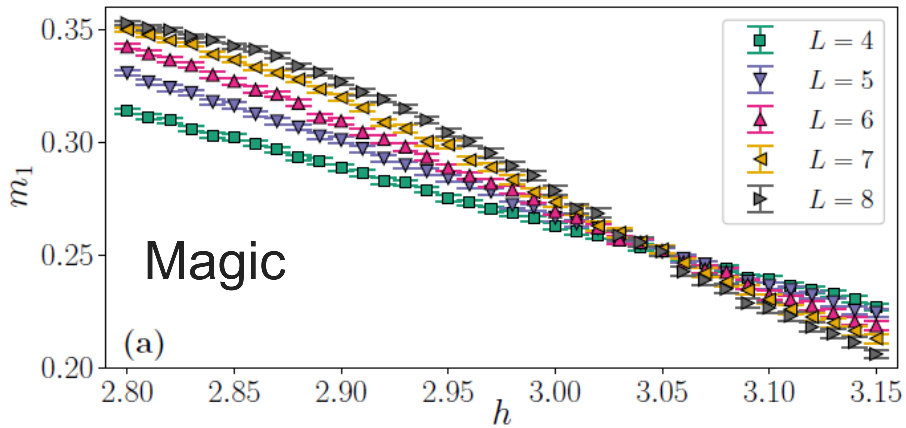
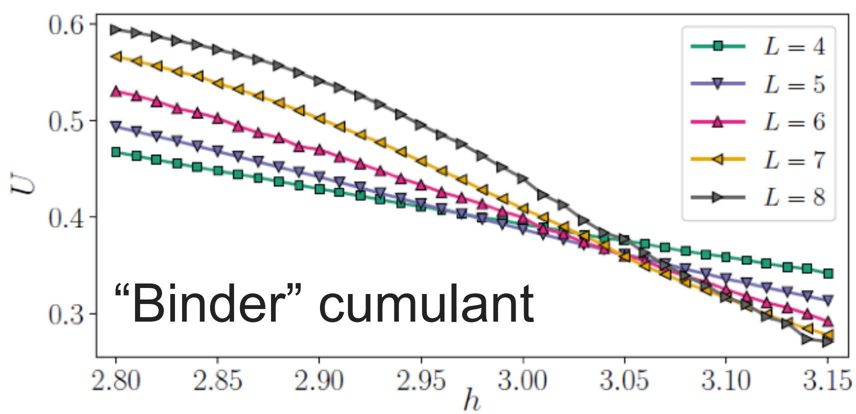




- at finite bond dimension, magic detects critical behavior *better* than the order parameter

Now, theory with matter - Schwinger model

- Quantum link formulation

Chandrasekharan and Wiese, 1994-97; Banerjee et al., PRL 2012

$$H = \sum_j [-t(\psi_j^\dagger S^+_{j,j+1} \psi_j + \mathrm{h.c.}) + V n_j n_{j+1} + U(-1)^j n_j]$$

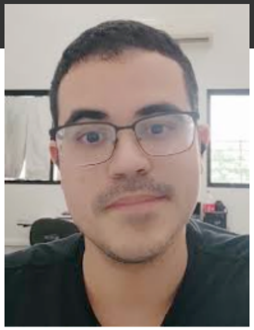
- Exact dual to a constrained spin model (“PXP”)

Surace et al., PRX 2020

$$\hat{H}_{\mathrm{FSS}} = \sum_j \left(\Omega\, \hat{\sigma}_j^x + \delta\, \hat{n}_j\right) + \sum_{j<l} V_{j,l}\, \hat{n}_j \hat{n}_l$$

Fendley, Sengupta, Sachdev, PRB2004;
Phase diagram: Chepiga and Mila,
Giudici et al., and more
Experiments: Lukin’s group

1D gauge theories and constrained spin models



See also: Smith et al., 2406.14348

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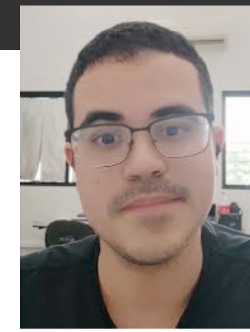
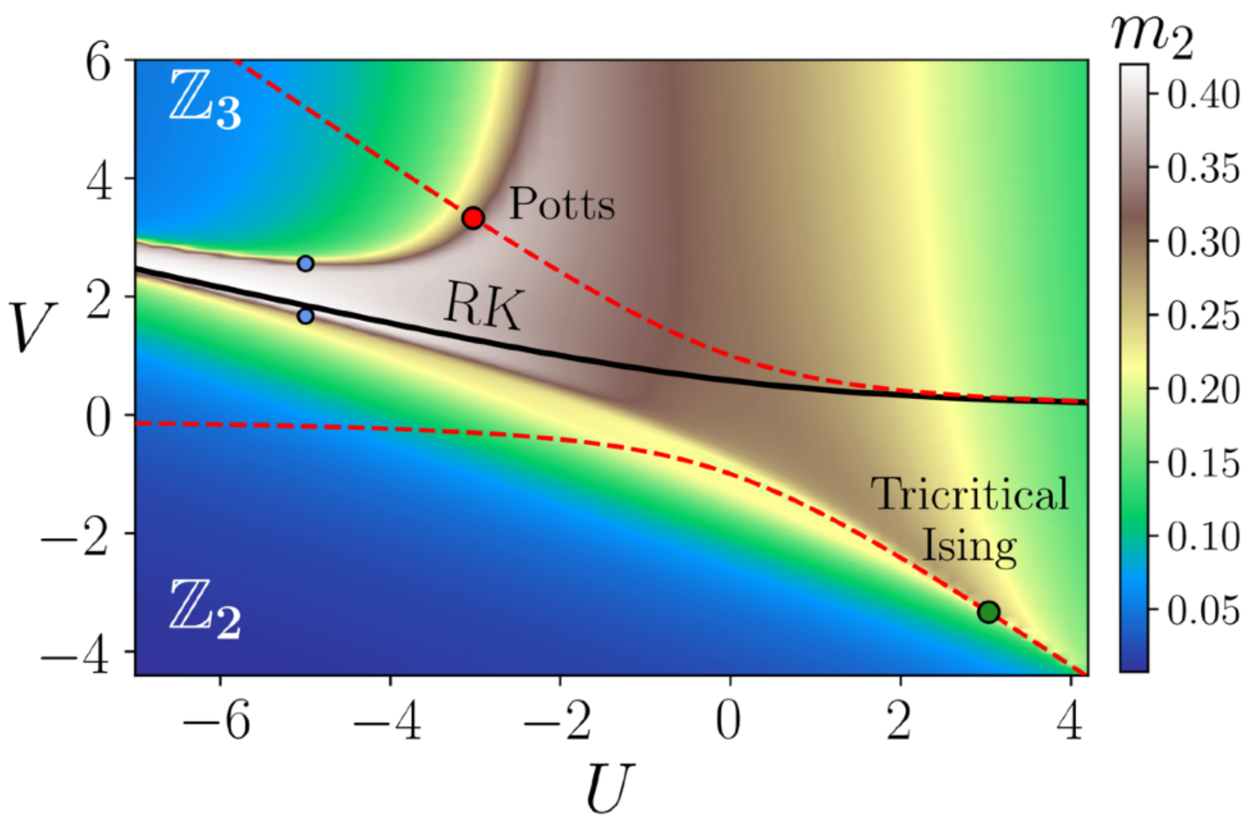


Magic density



Dashed: integrable lines. Thin: exactly soluble
L=30. Error < 10^-3

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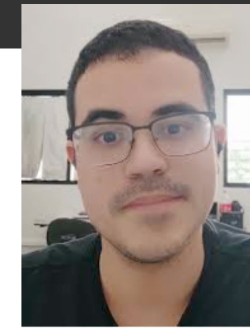
1D gauge theories and constrained spin models



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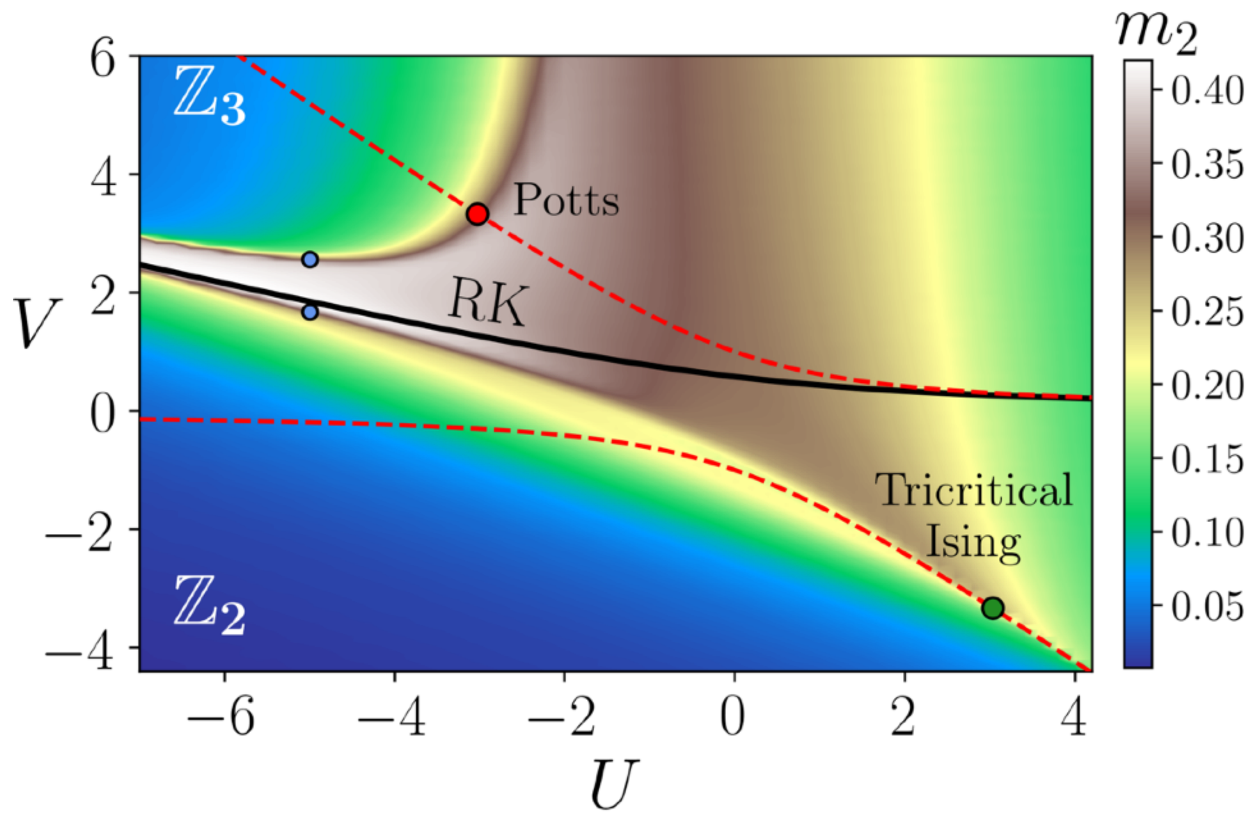

- Maximum typically inside phases
- Minimum inside ordered phases

Dashed: integrable lines. Thin: exactly soluble
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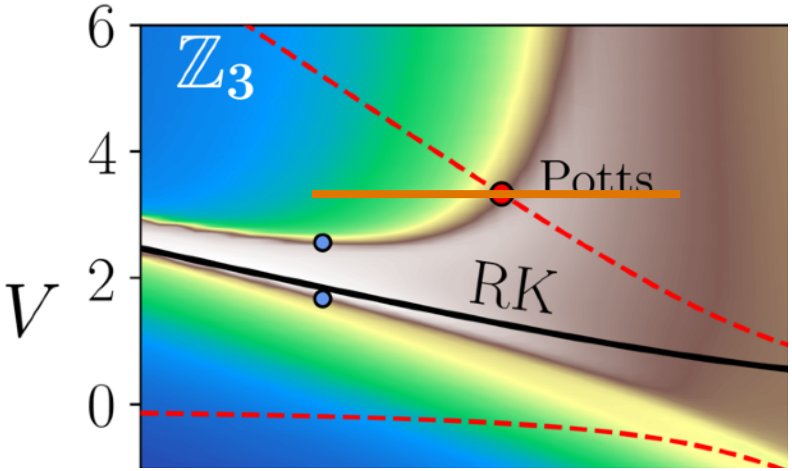
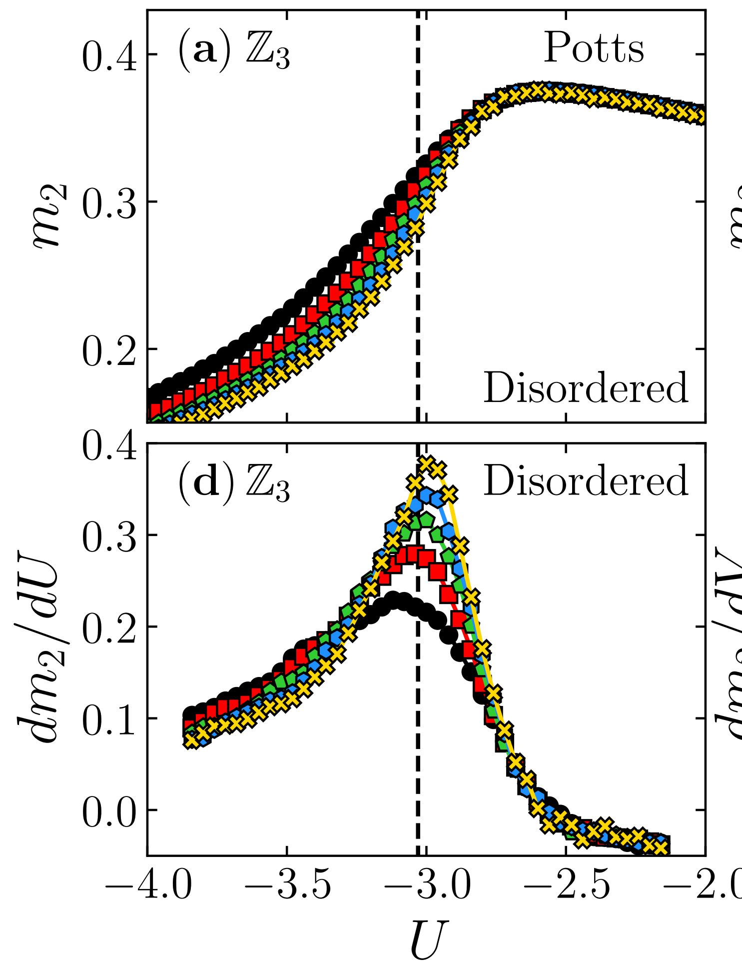
Magic approaching the continuum limit





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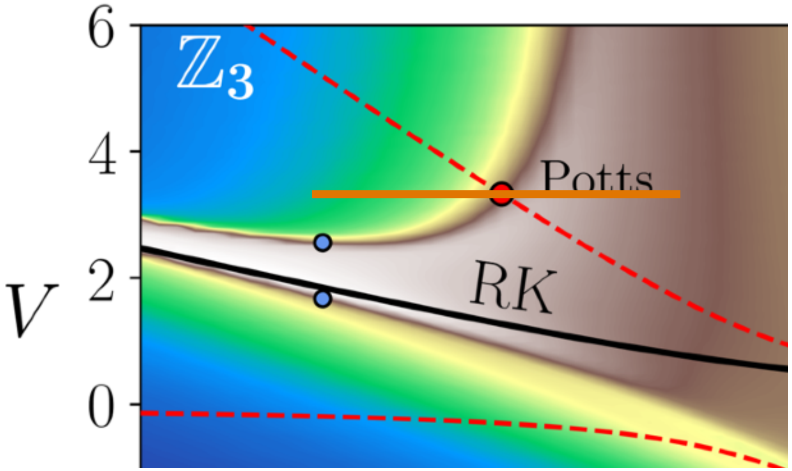
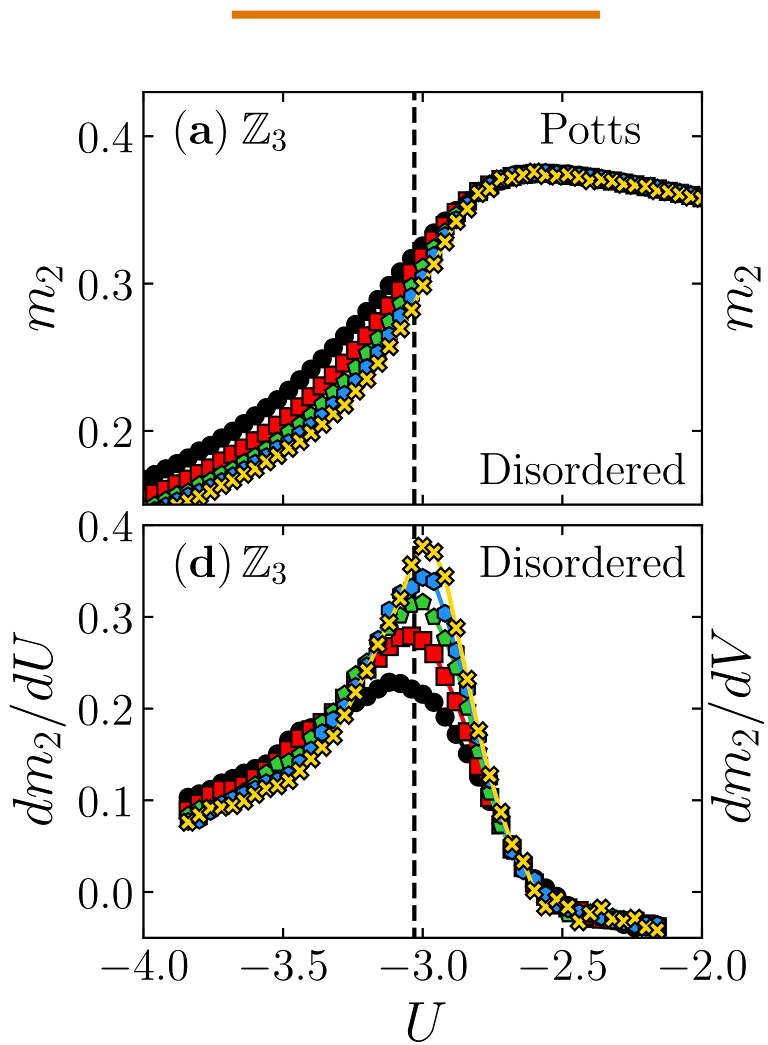
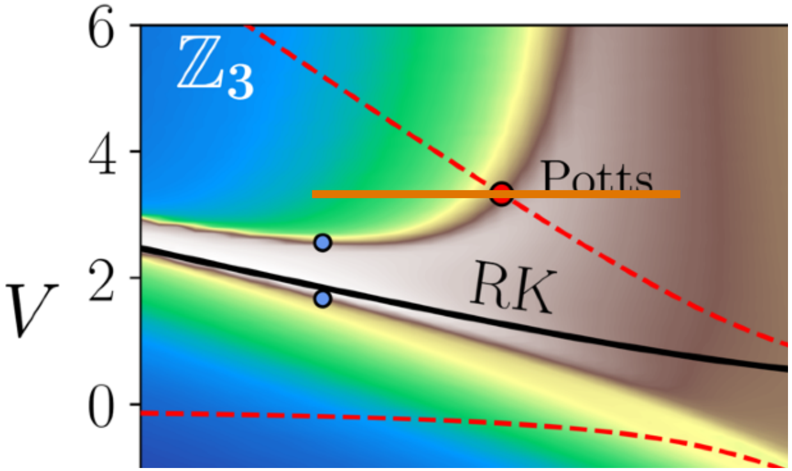
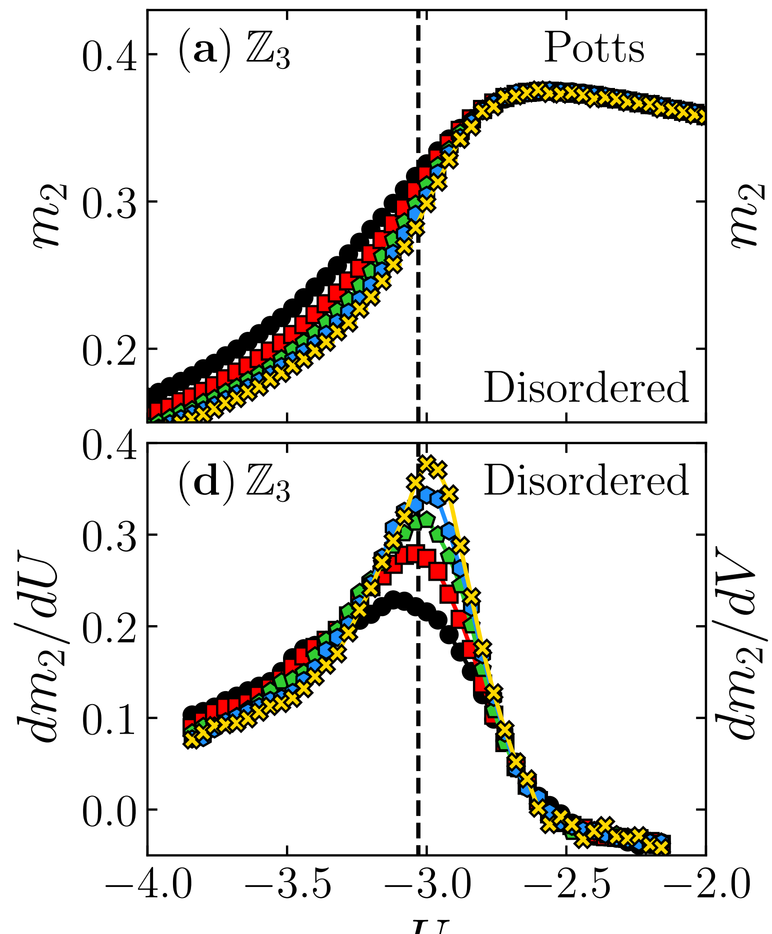




- Magic shows flexes at transition points

Magic approaching the continuum limit



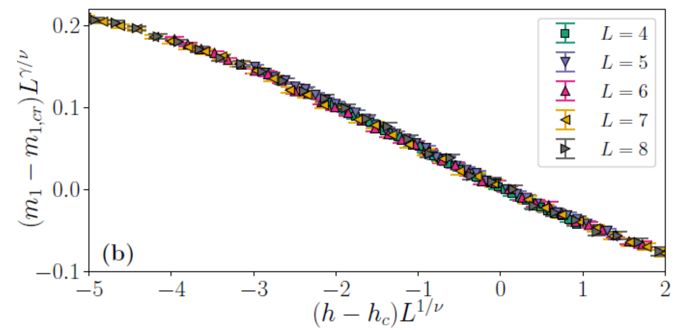


- Magic shows flexes at transition points
- Criticality unrelated to magic strength

What have we learned?

Q: Does magic relate to physical phenomena?

Yes! Both in 1D and 2D, full state magic diagnoses transition points (flexes), and displays universality. But importantly, one needs to look at derivatives!



This is already quite informative, even more than entanglement!

Few things I'd like to discuss today

Tensor network and magic: how to compute it

Compute

PRXQ 4, 040317 (2023), PRL 133, 010601 (2024)

Magic in lattice gauge theory: is there something interesting?

Learn

2409.01789

Can magic lead us to understand the deeper structure of many-body Hilbert spaces?

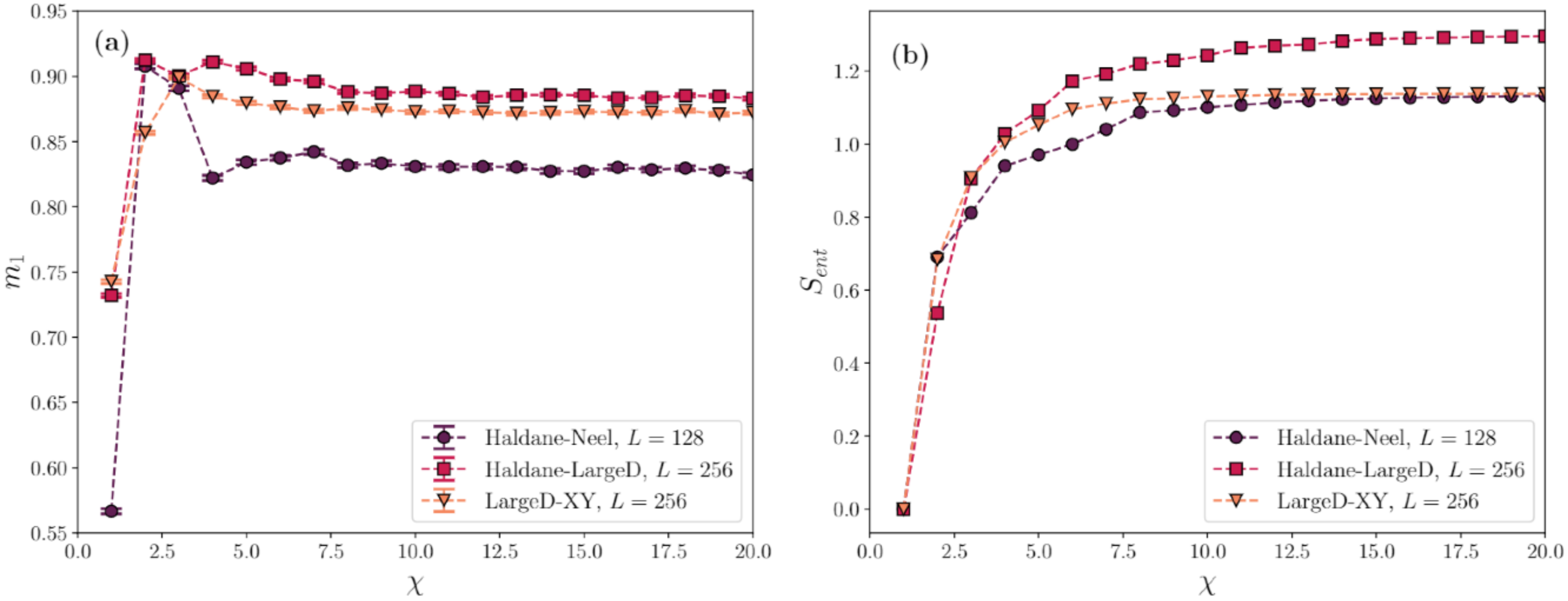
New methods?

PRB 110, 045101 (2024) and 2411.11720.

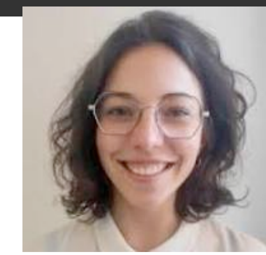
Magic vs entanglement in matrix product states

<u>Idea</u>: try to understand the relation between magic and separability in the context of simple states

Spin-1
$$H = \sum_{i=1}^{N} (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + J_Z S_i^z S_{i+1}^z) + D \sum_{i=1}^{N} S_i^{z2}$$

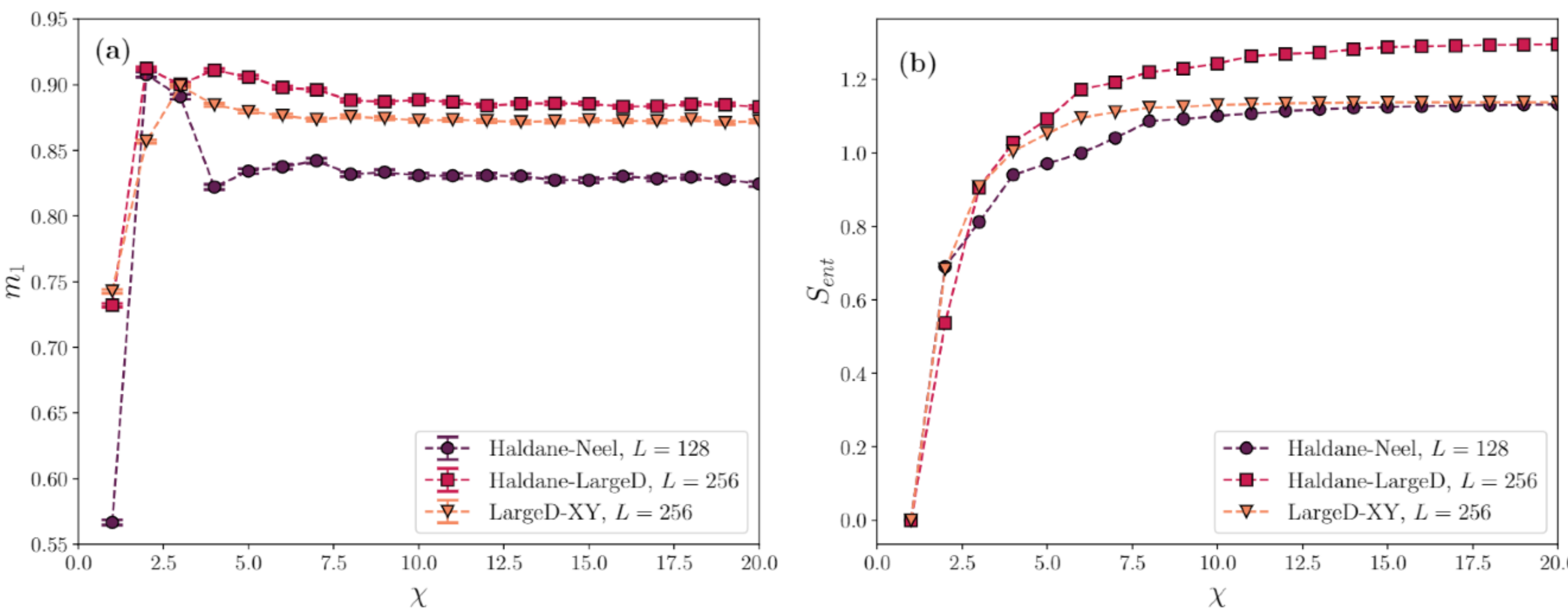


Magic vs entanglement in matrix product states

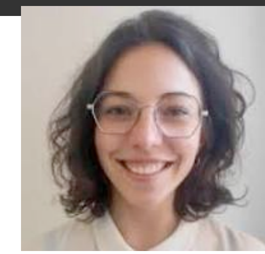


Idea: try to understand the relation between magic and separability in the context of simple states

Details of the model are unimportant!



Magic vs entanglement in matrix product states



Idea: try to understand the relation between magic and separability in the context of simple states

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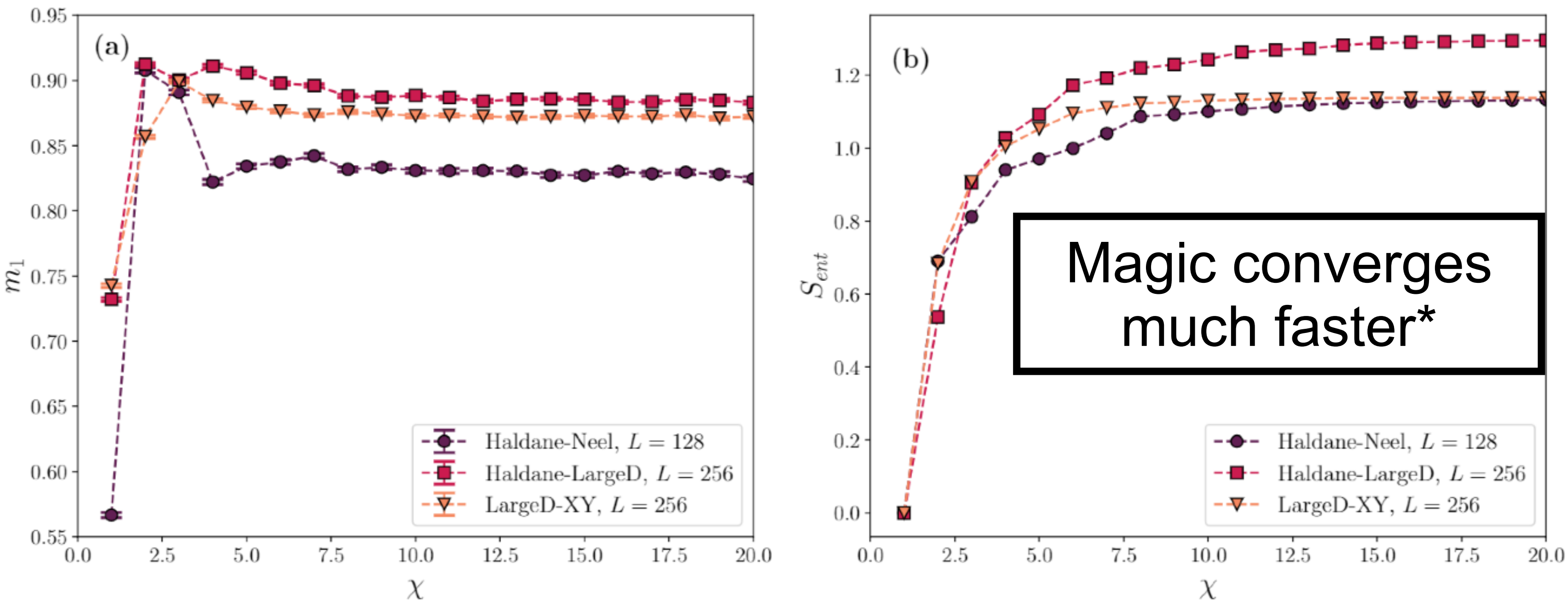


Magic converges much faster*

*note y-axis scale

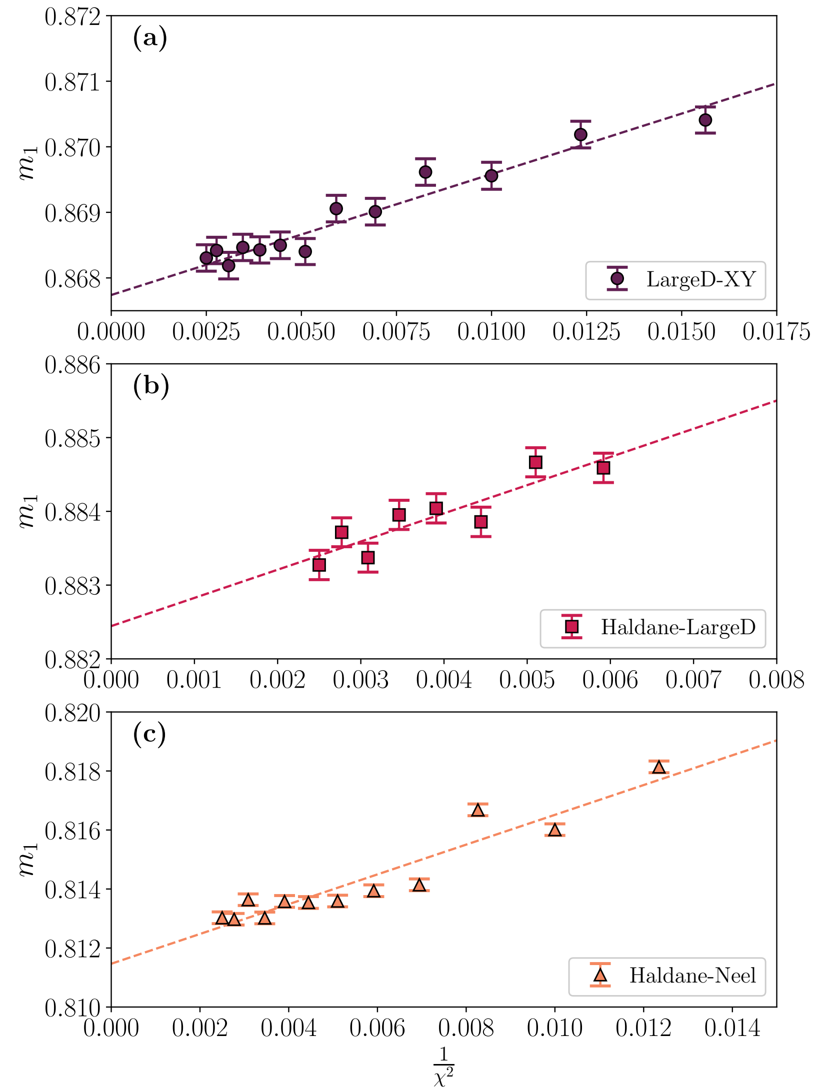
Magic vs entanglement in matrix product states

- In gapped phases, convergence is too fast to understand scaling
- At critical points, we always observe:

$$m_1(\chi) = m_1 + \frac{a_o}{\chi^2}$$

- Same scaling also true for random MPS (Lami et al., arXiv:2404.18751)

Clifford augmented MPS! arXiv:2405.09217



CAMPS: a novel class of variational states

Clifford augmented MPS! arXiv:2405.09217

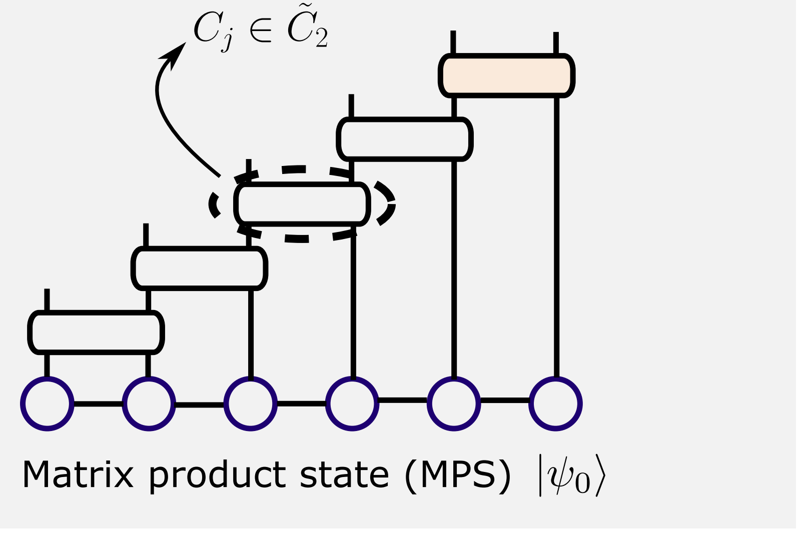
See also: stabilizer TN (2403.08724) and hybrid stabilizer MPOs (PRL 133, 150604 (2024))

- Fundamentally simple structure (local contractions)

- Can input in *arbitrary entanglement* at fixed cost

- Impressive performance gain for ground states!

Less clear for dynamics: 2407.01692; 2407.03202



CAMPS: a novel class of variational states?

Trades a constant overhead… for what?

1. Is there a new corner of the Hilbert space?
2. If yes, when are CAMPS overpowering MPS?

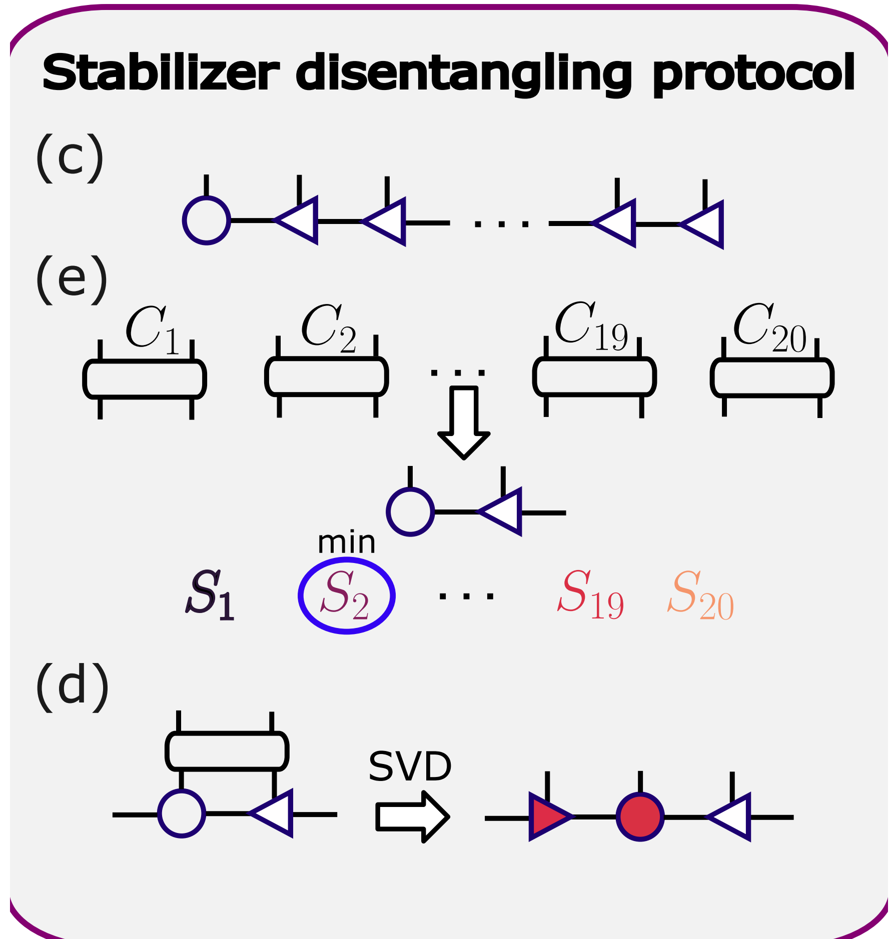
Reverse questions:

can we disentangle MPS with local Clifford??

Locally Clifford disentangler

Local Stabilizer disentangler: CAMPS cost is constant

Distematically removes entanglement, magic is invariant



Key methodological advance: number of gates is 20, not 720 as in 2405.09217

Is there a new corner of the Hilbert space?

Clifford disentangled state reduced density matrix

$$\rho_A^{(SM)} = \mathrm{Tr}_{\bar{A}} |\Psi[C_{\ell;L-\ell}]\rangle\langle\Psi[C_{\ell;L-\ell}]|$$

Its entropy (denoted as SMEE)

$$S_A^{(SM)} = -\mathrm{Tr}\rho_A^{(SM)} \ln \rho_A^{(SM)}$$

Entanglement gain:

$$\Delta(\ell_A; L) = S_A - S_A^{SM}$$

Is there a new corner of the Hilbert space?

Clifford disentangled state reduced density matrix

$$\rho_A^{(SM)} = \mathrm{Tr}_{\bar{A}} |\Psi[C_{\ell;L-\ell}]\rangle\langle\Psi[C_{\ell;L-\ell}]|$$

Its entropy (

Does Delta increase with size, or not?

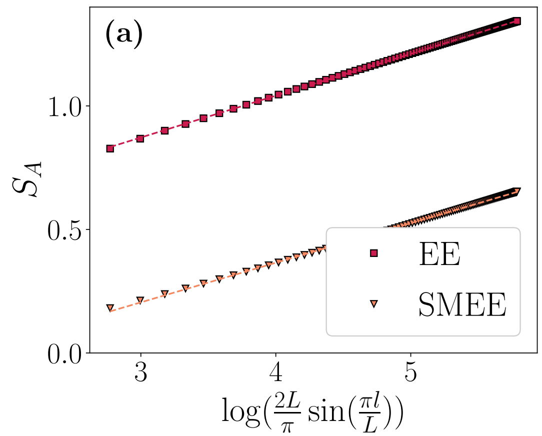
$$S_A = -\mathrm{Tr}\rho_A \ln \rho_A$$

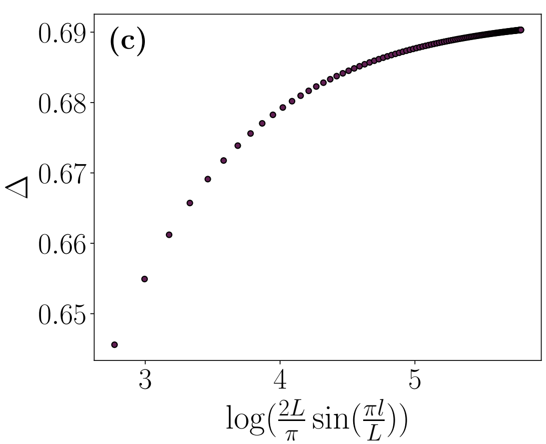
Entanglement gain:

$$\Delta(\ell_A; L) = S_A - S_A^{SM}$$

Is there a new corner of the Hilbert space? Look at conformal field theories!

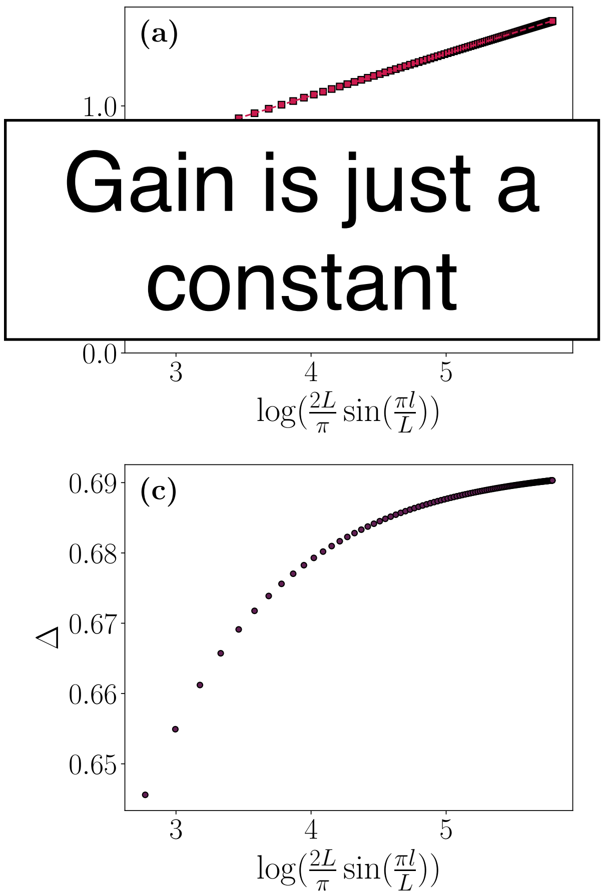
XXZ model





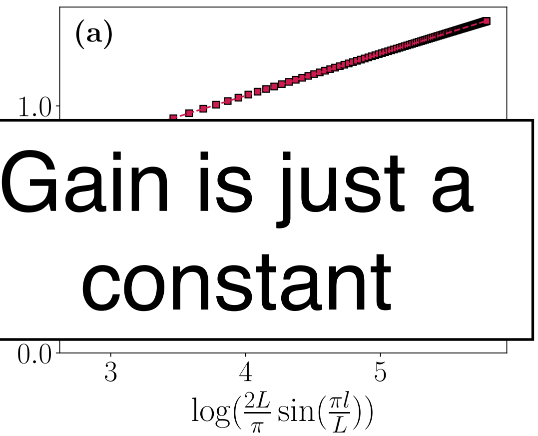
Is there a new corner of the Hilbert space? Look at conformal field theories!

XXZ model

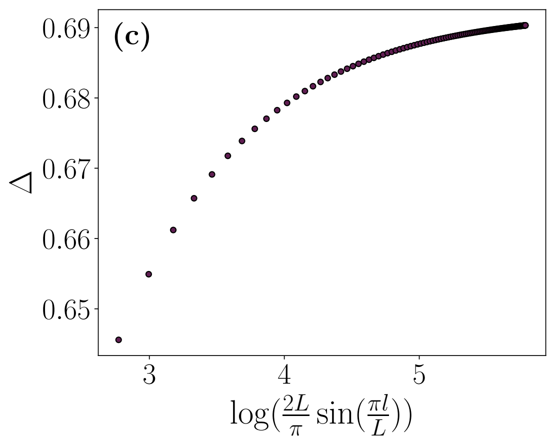


Is there a new corner of the Hilbert space? Look at conformal field theories!

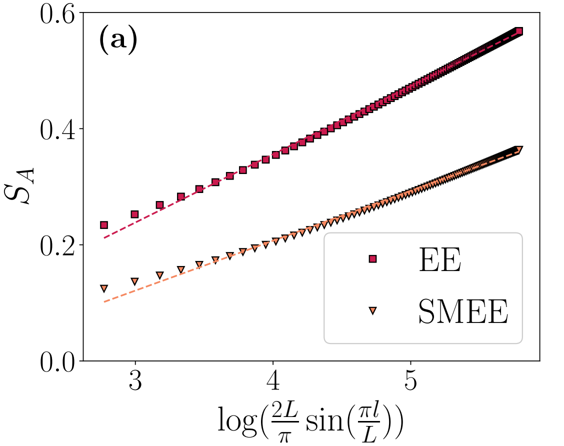
XXZ model

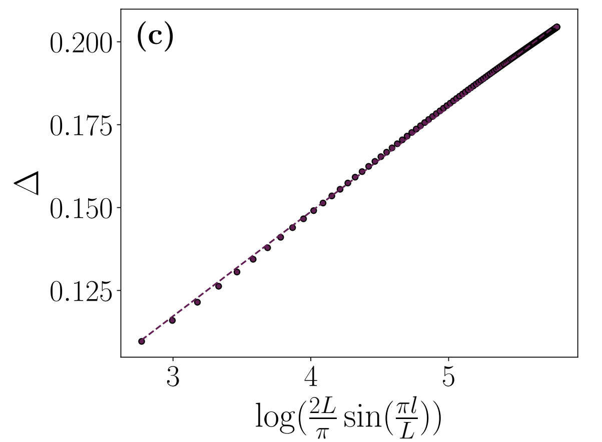


Gain is just a constant



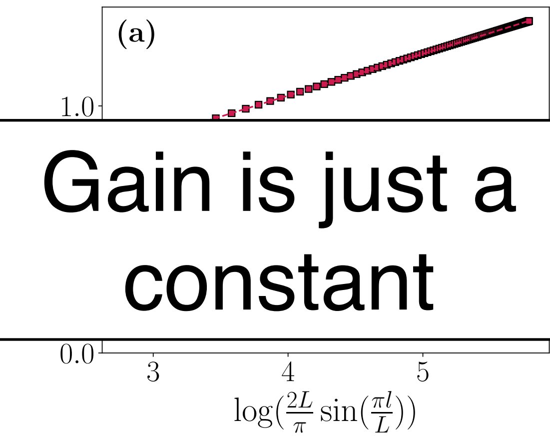
Tricritical Ising chain



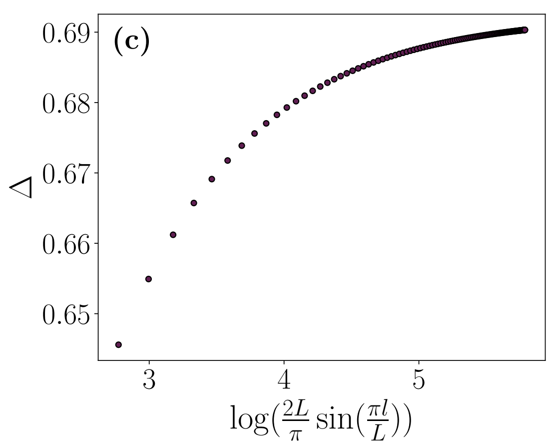


Is there a new corner of the Hilbert space? Look at conformal field theories!

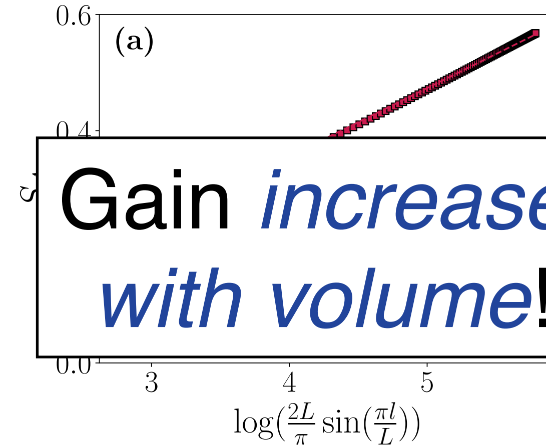
XXZ model



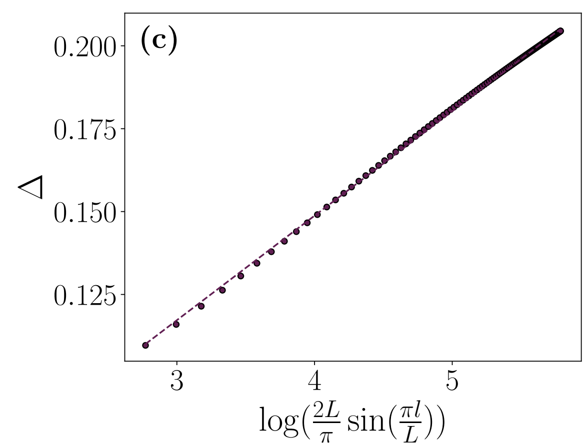
Gain is just a constant



Tricritical Ising chain

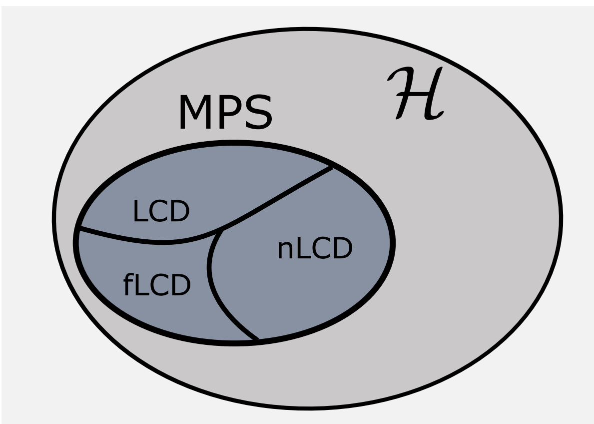


Gain *increases with volume*!!



CAMPS: a novel class of variational states?

1. Is there a new corner of the Hilbert space?



Local Clifford disentanglable (LCD)

$$\chi_{MPS} - \chi_{CAMPS} = O(L^{\beta})$$

2. If yes, when are CAMPS overpowering MPS?

When are CAMPS better?

Qualitative insight: CAMPS can decode away entanglement stored in non-local stabilizer degrees of freedom

Extreme case: only stabilizer has support in A+B

—> I shall be able to remove some entanglement between A and B by keeping magic constant

—> quantity to look at: mutual SRE

$$L_n(\rho_{AB}) = \tilde{M}_n(\rho_{AB}) - \tilde{M}_n(\rho_B) - \tilde{M}_n(\rho_A)$$

What is mutual SRE monitoring

$$L_n(\rho_{AB}) = \tilde{M}_n(\rho_{AB}) - \tilde{M}_n(\rho_B) - \tilde{M}_n(\rho_A)$$

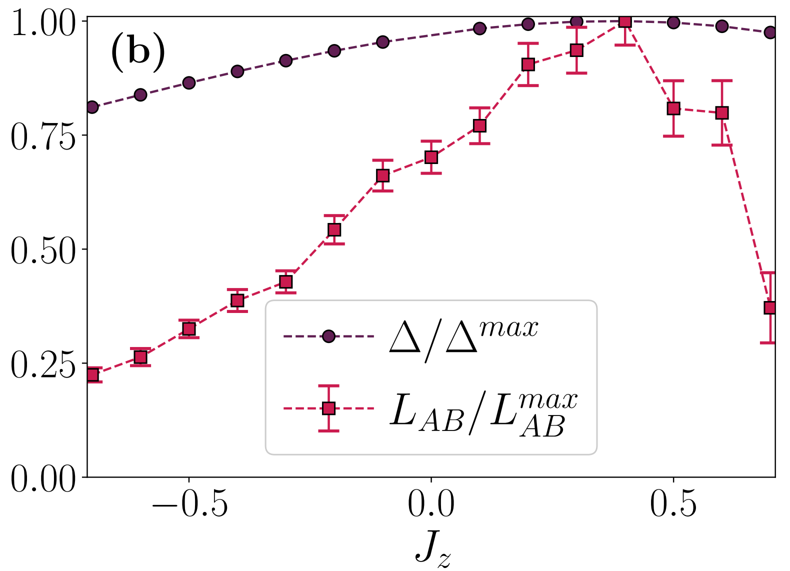
Not a measure of magic in general (at least, not for qubits)

Still, very important! Participation entropy in stabilizer space (Turkeshi, Schiro',…)

If **mutual SRE<0**, there could be some shared qubit that can be disentangled via Clifford only!

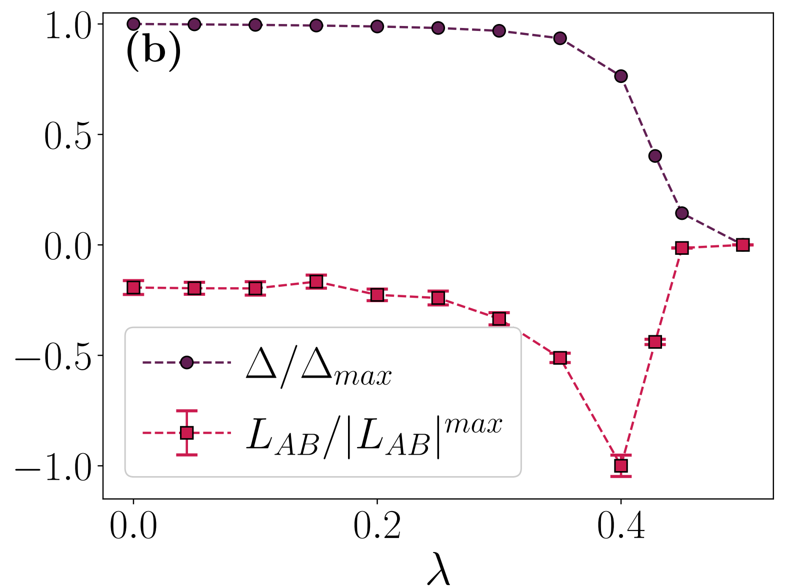
Can mSRE distinguish?

XXZ model



mSRE always positive
(constant gain)

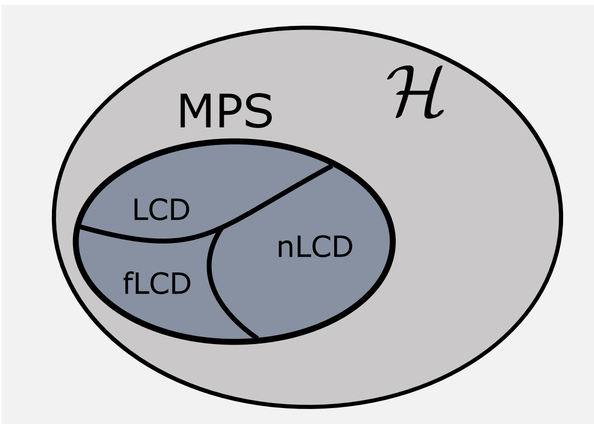
Tricritical Ising chain



mSRE negative at
criticality!
(Increasing gain)

CAMPS: a novel class of variational states?

1. Is there a new corner of the Hilbert space?



Local Clifford disentanglable (LCD)

$$\chi_{MPS} - \chi_{CAMPS} = O(L^{\beta})$$

2. If yes, when are CAMPS overpowering MPS?

CAMPS systematically better if mutual SRE becomes negative!! Non-local stabilizer (anti-) correlations

Analytical understanding: Cluster Ising models

$$H_1 = -\sum_{i=1}^{L-2} (S_i^x S_{i+1}^z S_{i+2}^x) + h \sum_{i=1}^{L} S_i^z \ .$$

c=1 critical point feature an exact self-duality

$$H = \sum_{s=1}^{2} H_{\mathrm{Ising}}^{s}$$

- The duality transformation is a Clifford transformation
- Of course, mutual SRE <0! (Hidden Stabilizer info encoded)

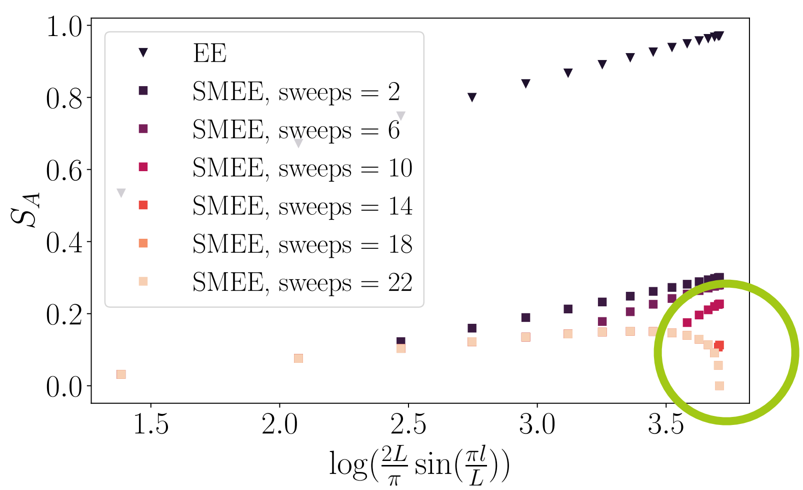
Smacchia, Pascazio, Fazio, Amico, .. PRA 2011

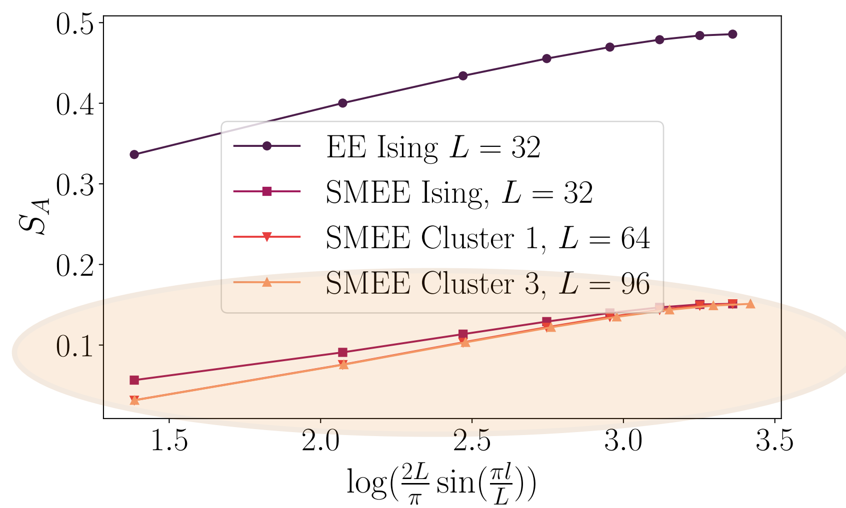
Analytical understanding: Cluster Ising models

Predictions:

(A) At l=L/2, SMEE *vanishes*

(B) For l<L/2, SMEE coincide with that of a Clifford disentangled Ising chain (central charge from 1 to 1/2)





Open questions

We are barely scratching the surface of the interface magic/many-body systems!

Simply, too many open questions to list, here some:

Magic and deeper entanglement structures? (Modular Hamiltonian complexity)

Which 2D models can CAMPS solve that MPS cannot?

What about many-body with true measures (robustness)??

More on gauge theories? Dynamics? (See talk by Martin)

P. Tarabunga

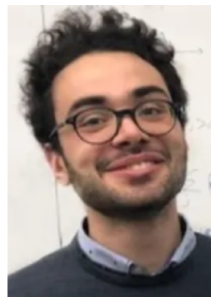
T. Chanda

M. Frau

E. Tirrito

G. Fux

S. Fazio



S. Oliviero

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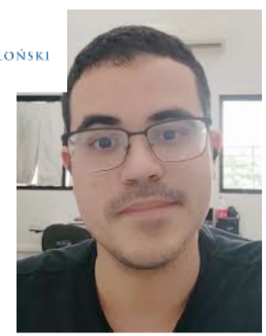
A. Hamma

G. Lami

M. Collura

J. Zakrzewski



P. Falcao

MC. Bañuls

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ICTP


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Thank you!

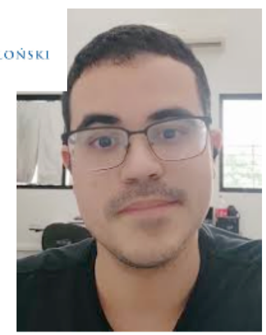
SISSA


G. Lami


M. Collura


J. Zakrzewski

UNIWERSYTET JAGIELLOŃSKI W KRAKOWIE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